Using differences to identify patterns

Answers

Practice Problems

1. For problems a – d, find the next THREE numbers in the pattern and then write a RULE for the pattern.

(a) 15, 17, 19... 21, 23, 25

$2n+13$

(b) 21, 24, 27... 30, 33, 36

$3n+18$

(c) 87, 82, 77... 72, 67, 62

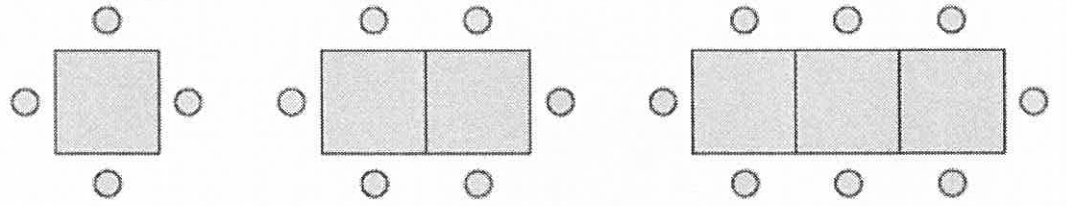
$-5n+92$

(d) 2, 4, 8, 16... 32, 64, 128

2^n

2. Use the desk pattern below to fill in the table and answer the questions below.

Arranging desks



(a) Complete the table

Number of desks (D)	1	2	3	4	5	6
Number of chairs (C)	4	6	8	10	12	14

(b) What is the RULE for this pattern?

$2n+2$

(c) How many chairs will there be when you have 15 desks in the room?

$2(15)+2 = 32$

3. Write the rule for the expression that has the following pattern: 6, 11, 16, 21....

$5n+1$

4. A woman deposits \$100 into her son's savings account on his first birthday. On his second birthday she deposits \$125, \$150 on his third birthday, and so on.

(a) How much money would she deposit into her son's account on his **17th** birthday?

$U_{17} = 100 + (17-1)25$ $\boxed{U_{17} = 500}$

(b) How much in **total** would she have deposited after her son's 17th birthday?

$S_{17} = \frac{17}{2}(2(100) + (17-1)25)$ $\boxed{S_{17} = 5100}$

5. A National Lottery is offering prizes in a new competition. The winner may choose one of the following.

Option one: \$1000 **each week** for 10 weeks.

Option two: \$250 in the first week, \$450 in the second week, \$650 in the third week, **increasing by \$200** each week for a total of 10 weeks.

Option three: \$10 in the first week, \$20 in the second week, \$40 in the third week continuing to double for a total of 10 weeks.

(a) Calculate the amount you receive in the **tenth week**, if you select

(i) **option two**;

$U_{10} = 250 + (10-1)200$

$\boxed{U_{10} = 2050}$

(ii) **option three**.

$U_{10} = 10(2)^{10-1}$

$\boxed{U_{10} = 5120}$

(b) What is the total amount you receive if you select **option two**?

$S_{10} = \frac{10}{2}(2(250) + (10-1)200)$ $= 11500$

(c) Which option has the greatest total value? Justify your answer by showing all appropriate calculations.

(1) 1000×10

$\boxed{10000}$

(2) 11500

(3) $S_{10} = \frac{10(2^{10}-1)}{2-1}$

$\boxed{S_{10} = 10230}$

Option Two

Evaluating Expressions

Practice Problems

Question	Expression	X = -2	X = 0	X = 2
5	2x - 3	-7	-3	1
6	4 – 3x	10	4	-2

1. $15 + 5\,(3 + 2)$ — 40
2. $8 \cdot 12 \div 2 + 4$ — 52
3. $4\,(7 + 3 \cdot 9)$ — 136
4. $\frac{24 - 18}{3} + 2$ — 4
5. $4\,[3\,(8 - 5) + 6\,] - 3^2$ — 51

6. $5a + 7 + 6a + 5$

$11a + 12$

7. $-5x - 7y + 8 - 7x + 3x^2$

$3x^2 - 12x - 7y + 8$

8. $7y - 3(-4y - 7x + 6) + 4x$

$7y + 12y + 21x - 18 + 4x$

$25x + 19y - 18$

Solving Equations

Practice Problems

1. $\frac{x}{5} + 6 = 36$

$x = 150$

2. $-4x - 6 = -18$

$x = 3$

3. $6(2a - 4) = -36$

$12a - 24 = -36$

$a = -1$

4. $8c + 2 = 3c - 23$

$\frac{5c}{5} = \frac{-25}{5}$

$c = -5$

5. $3(-2n - 4) = -(6n + 12)$

$-6n - 12 = -6n - 12$

$0 = 0$

Infinite solutions

6. $2(d - 6) - 5 = 9(d + 3) + 5$

$2d - 12 - 5 = 9d + 27 + 5$

$\frac{7d}{7} = \frac{49}{7}$

$d = 7$

7. $5(m + 1) + 6 = 3(4 + m) + (2m - 3)$

$5m + 5 + 6 = 12 + 3m + 2m - 3$

$0m = -2$

No solution

8. Solve for t. $I = PRT$

$T = \frac{I}{PR}$

11. $\left(\frac{3x-5}{2} - \frac{2x+1}{3} = \frac{7}{12}\right)12$

$6(3x - 5) - 4(2x + 1) = 7$

$18x - 30 - 8x - 4 = 7$

$\frac{10x}{10} = \frac{41}{10}$

$x = 41/10$

Linear Functions

Practice Problems

Find the slope given the two points.

1. $(-3, 4)$ and $(-3, 6)$

Undefined

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{-3--3} = \frac{2}{0}$$

2. $(-6, -5)$ and $(-3, 1)$

$$\frac{1--5}{-3--6} = \frac{6}{3} = 2$$

3. $(2, 5)$ and $(6, 5)$

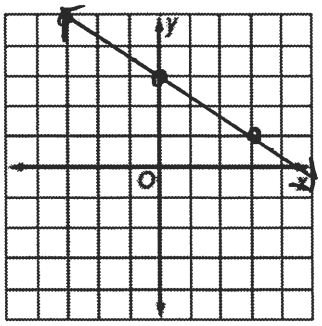
$$\frac{5-5}{6-2} = \frac{0}{4} = 0$$

4. $(-5, 7)$ and $(1, 2)$

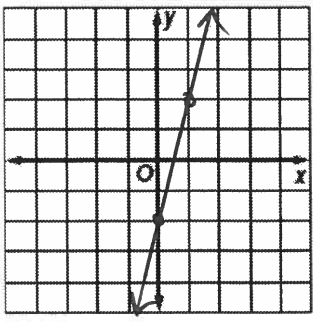
$$\frac{2-7}{1--5} = \frac{5}{6}$$

Graph each line below.

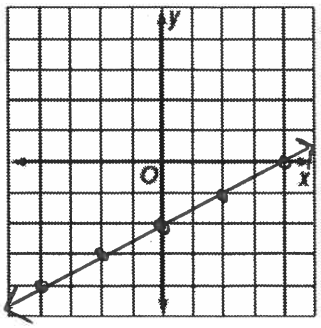
$y = -\frac{2}{3}x + 3$



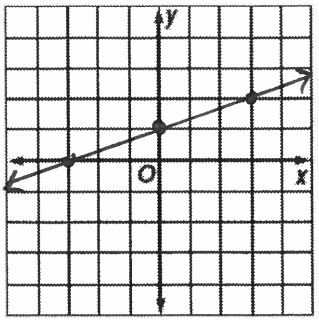
$-8x + 2y = -4$ $y = 4x - 2$



$y = \frac{1}{2}x - 2$



$y = \frac{1}{3}x + 1$



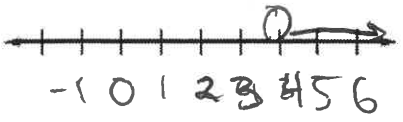
Inequalities

Practice Problems

Solve the following inequalities and graph them on the number line provided.

1. $5n + 2 > 22$

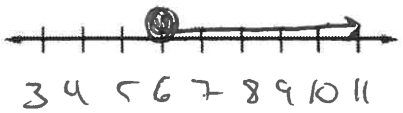
$n > 4$



2. $2x + 6 \leq 5x - 12$

$\frac{-3x}{-3} \geq \frac{-18}{-3}$

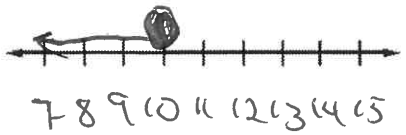
$x \geq 6$



3. $-3m + 7 \geq -23$

$-3m \geq -30$

$m \leq 10$

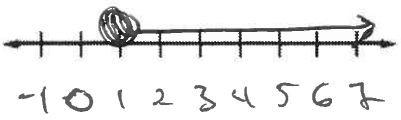


4. $-2(3x - 5) \geq 4(-5x + 6)$

$-6x + 10 \geq -20x + 24$

$\frac{14x}{14} \geq \frac{14}{14}$

$x \geq 1$



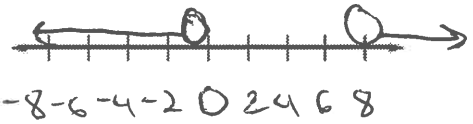
5. $4z + 7 < 5$ or $2z - 4 > 12$

$4z < -2$

$z < -\frac{1}{2}$

$2z > 16$

$z > 8$



Systems of Equations

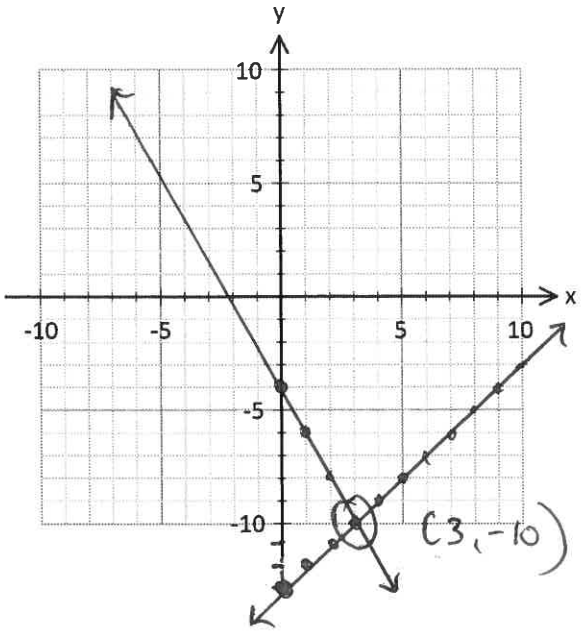
Practice Problems

1. Graph the following system on the Cartesian plane on the right.

$x - y = 13$
$2x + y = -4$

$y = x - 13$

$y = -2x - 4$



2. Solve the following systems of equations

a $y = x - 3$
$y = 1 - x$

(2, -1)

$x = 2$
$y = -1$

b $x - y - 1 = 0$
$y = 2x$

(-1, -2)

$x = -1$
$y = -2$

c $4x + 3y + 12 = 0$
$x - 2y + 3 = 0$

(-3, 0)

$x = -3$
$y = 0$

3. Solve the following question **algebraically** and **graphically**

$y = 2x - 5$ and $y = x^2 + 4x - 5$

$x^2 + 4x - 5 = 2x - 5$

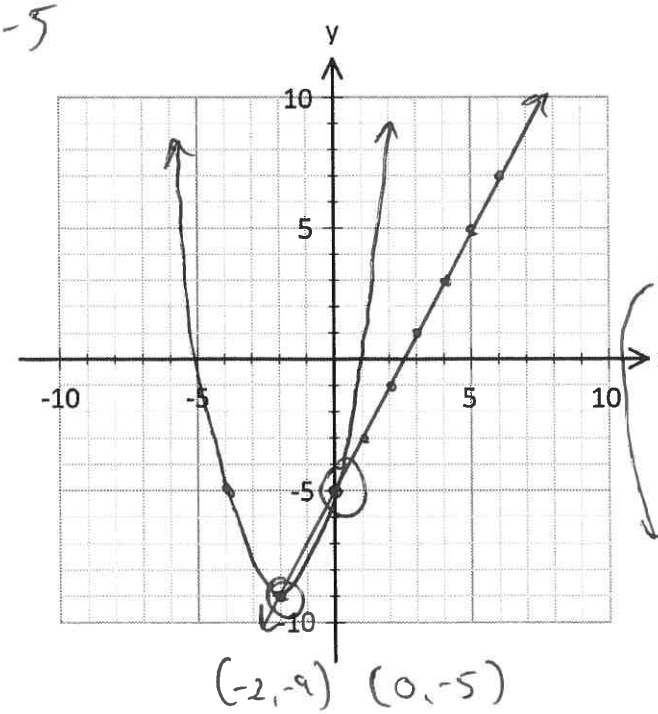
A B
$x^2 + 2x = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(0)}}{2(1)}$

$x = \frac{-2 + 2}{2}$ $x = \frac{-2 - 2}{2}$

$x = 0$ $x = -2$



$y = 2(0) - 5$
$= -5$

$y = 2(-2) - 5$
$= -9$

(0, -5)

(-2, -9)

The Rules of Exponents

Practice Problems

1. Simplify the following exponents.

(a) $x^2 \bullet x^5$ (b) $\frac{m^7}{m^3}$ (c) $(a^5)^6$ (d) b^{-2} (e) y^0

x^7 m^4 a^{30} $\frac{1}{b^2}$ 1

2. Write the expression with positive exponents. $a^{-4}b^3c^{-2}$

3. Simplify. $\left(\frac{4a^2bc^3}{5a^6bc}\right)^3$ $\left(\frac{4c^2}{5a^4}\right)^3 = \frac{64c^6}{125a^{12}}$

4. Simplify. $\frac{9x^{-5}y^3z^2}{27x^2y^{-4}z}$ $\frac{1x^{-7}y^7z}{3} = \frac{1y^7z}{3x^7}$

5. Simplify. $\left(\frac{4x^4y^5z^3}{2x^2y^6z^2}\right)^{-2}$ $\left(\frac{2xz}{y}\right)^{-2} = \frac{y^2}{4x^2z^2}$

6. Solve. $9^{2x+1} = 3^{x+5}$

$3^{2(2x+1)} = 3^{x+5}$

$4x+2 = x+5$

$\frac{3x}{3} = \frac{3}{3}$ $x=1$

7. Solve. $4^{-x+1} = 8^{2x-2}$

$2^{2(-x+1)} = 2^{3(2x-2)}$

$-2x+2 = 6x-6$

$-8x = -8$ $x=1$

Expanding Brackets and Factoring

Practice Problems

EXPAND BRACKETS to simplify the expression.

1. $(x + 5)(x + 3)$ $x^2 + 8x + 15$

2. $(a + 7)(a - 3)$ $a^2 + 4a - 21$

3. $(2x - 7)(3x + 5)$ $6x^2 - 11x - 35$

Find the roots, zero's or x intercepts of each expression using the Quadratic formula.

4. $t^2 - 11t + 24$ $t = 8$ $t = 3$

5. $n^2 + n - 42$ $n = -7$ $n = 6$

6. $8n^2 - 36n + 40$ $n = \frac{5}{2}$ $n = 2$

7. $36m^2 - 49$ $m = -\frac{7}{6}$ $m = \frac{7}{6}$

Quadratics

Practice Problems

For each problem, complete a table of values. Find the intercepts, axis of symmetry, and coordinates of the vertex. Then graph the quadratic.

1. $x^2 - 2x - 8$

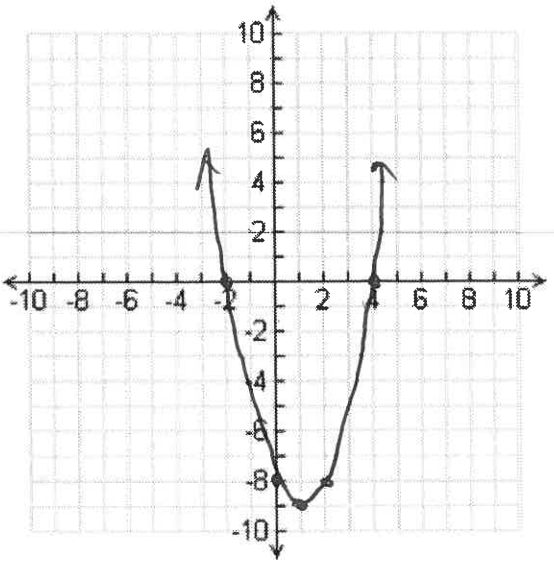
X	Y
-2	0
-1	-4
0	8
1	9
2	8
3	4
4	0

x-intercepts: $(-2, 0)$ $(4, 0)$

axis of symmetry: $x = 1$

Coordinates of the vertex: $(1, -9)$

y-intercept: $(0, -8)$



2. $(x-2)^2$

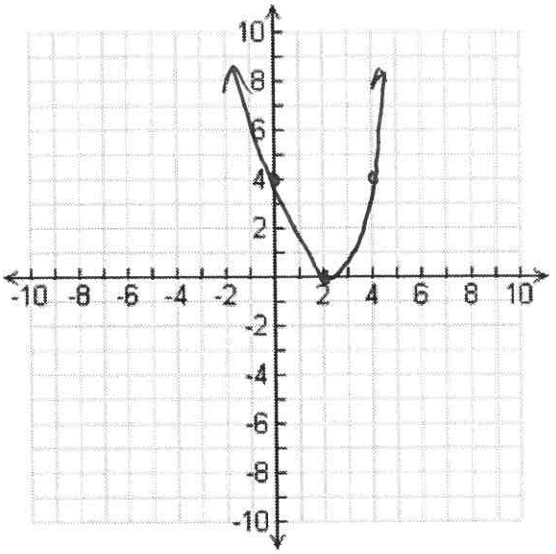
X	Y
-1	9
0	4
1	1
2	0
3	1
4	4
5	9

x-intercepts: (2,0)

axis of symmetry: $x = 2$

Coordinates of the vertex: (2,0)

y-intercept: (0,4)



3. $-x^2-2x+3$

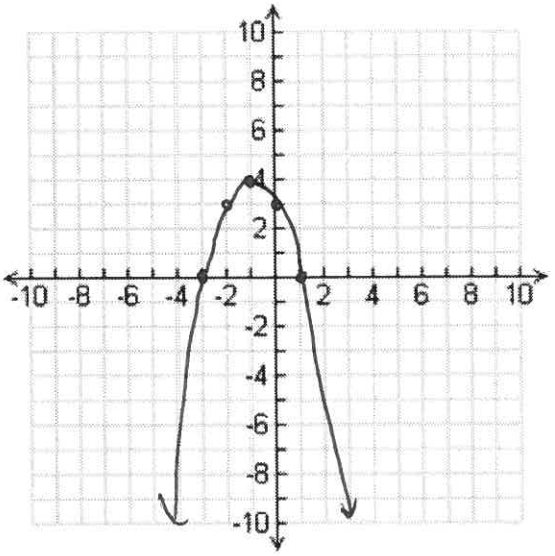
X	Y
-3	0
-2	3
-1	4
0	3
1	0
2	-5
3	-12

x-intercepts: (-3,0) (1,0)

axis of symmetry: $x = -1$

Coordinates of the vertex: (-1,4)

y-intercept: (0,3)



4. $-(x-4)(x+2)$

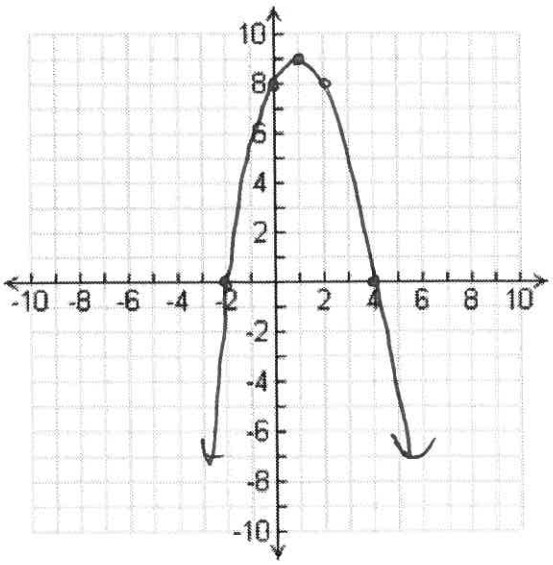
X	Y
-2	0
-1	5
0	8
1	9
2	8
3	5
4	0

x-intercepts: (-2,0)(4,0)

axis of symmetry: $x = 1$

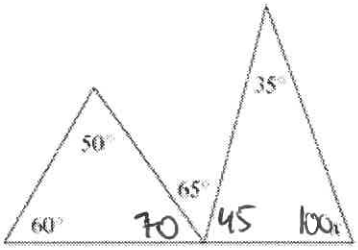
Coordinates of the vertex: (1,9)

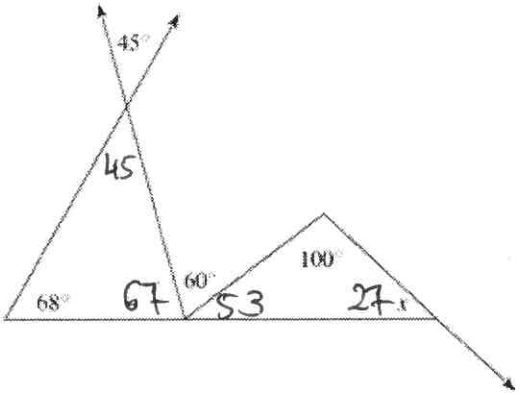
y-intercept: 8



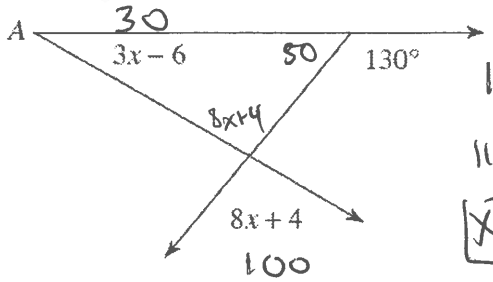
Geometry and Pythagoras

1) Solve the following for ***x***.





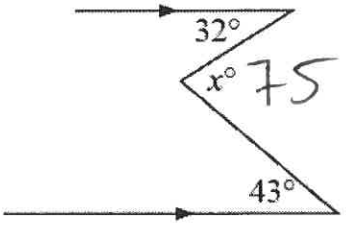
$8x+4+3x-6+50=180$

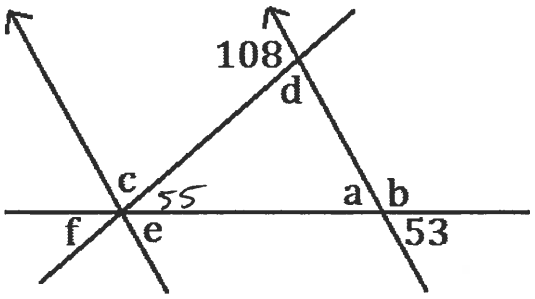


$11x+48=180$

$11x=132$

$x=12$





A = 53

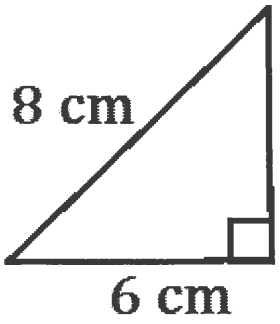
B = ~~1~~27

C = 72

D = 72

E = 72

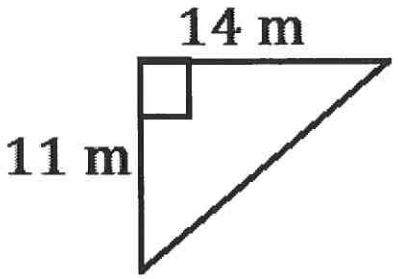
2) Find the missing side of the triangles below.



$8^2 - 6^2 = a^2$

$28 = a^2$

$5.29 = a$

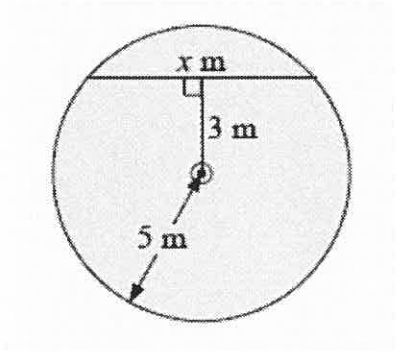


$11^2 + 14^2 = c^2$

$317 = c^2$

$17.8 = c$

3) Find the values of **X**.



$5^2 - 3^2 = b^2$

$25 - 9 = b^2$

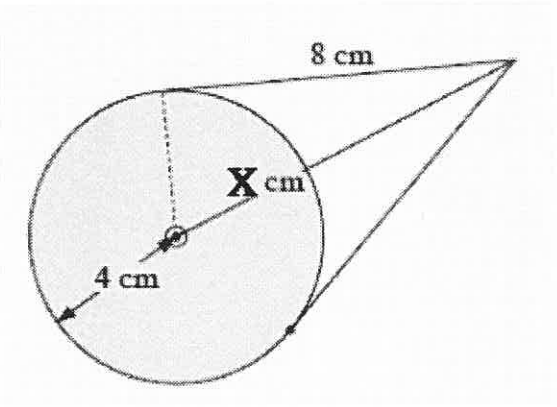
$16 = b^2$

$4 = b$

$X = 4 + 4$

$X = 8\text{m}$

4) Find the values of **X**.



$8^2 + 4^2 = c^2$

$64 + 16 = c^2$

$\sqrt{80} = \sqrt{c^2}$

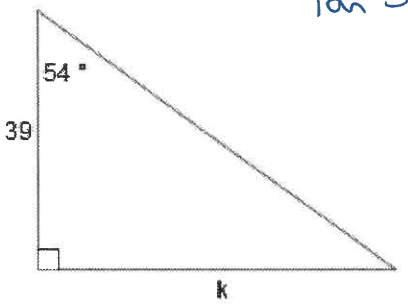
$8.94^{cm} = c$

Trigonometry

Find the side indicated by the variable. Round to the nearest tenth.

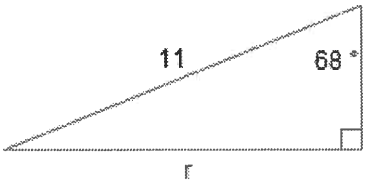
1) k = 53.7

$\tan 54 = \frac{k}{39}$



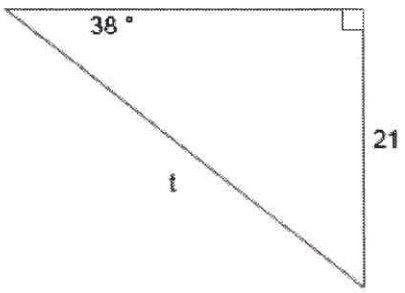
2) r = 10.2

$\sin 68 = \frac{r}{11}$



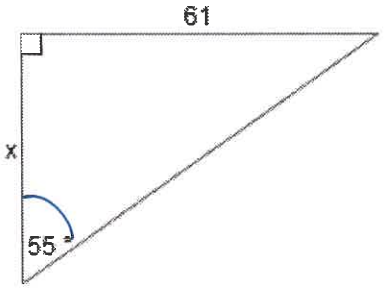
3) t = 34.1

$\sin 38 = \frac{21}{t}$



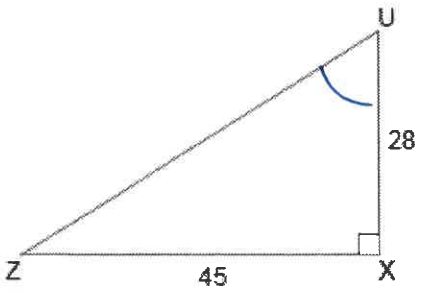
4) x = 42.7

$\tan 55 = \frac{61}{x}$



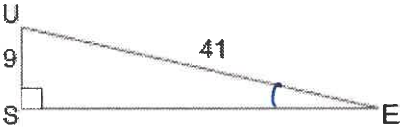
Find the measure of the indicated angle to the nearest degree.

1) m∠U = 58.1 °



$\tan^{-1}\left(\frac{45}{28}\right)$

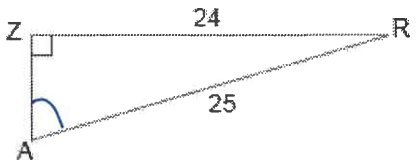
2) m∠E = 12.7 °



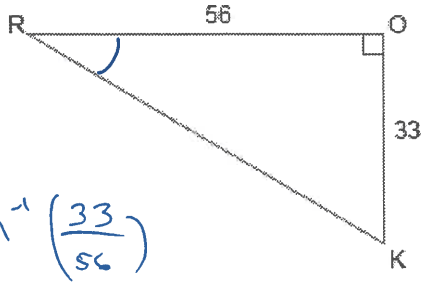
$\sin^{-1}\left(\frac{9}{41}\right)$

3) m∠A = 73.7 °

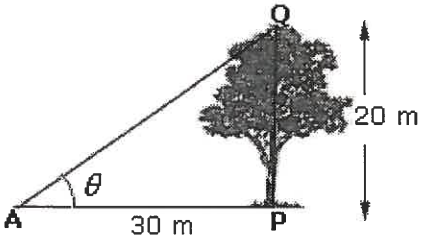
$\sin^{-1}\left(\frac{24}{25}\right)$



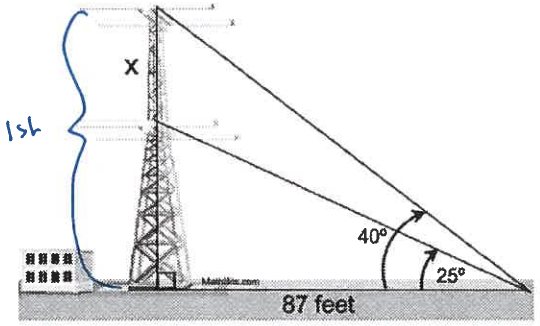
4) m∠R = 30.5 °



$\tan^{-1}\left(\frac{33}{56}\right)$



$\tan^{-1}\left(\frac{20}{30}\right) = 33.7°$



$\tan 40 = \frac{x}{87}$ $\quad$ $\tan 40 = \frac{x}{25}$

$x = 73$ $\quad$ $x = 21.0$

$73 - 21 = 52$ feet

1. José stands 1.38 kilometres from a vertical cliff.

(a) Express this distance in metres.

1380 m

José estimates the angle between the horizontal and the top of the cliff as 28.3° and uses it to find the height of the cliff.

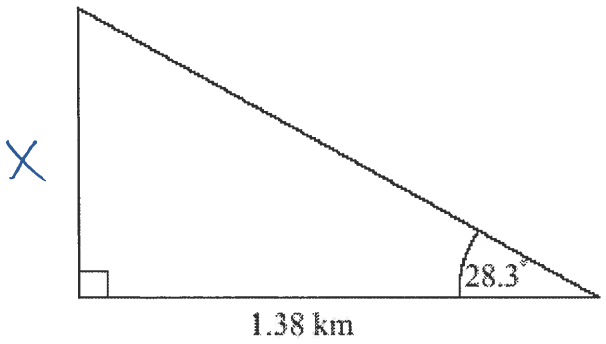


diagram not to scale

(b) Find the height of the cliff according to José's calculation. **Express your answer in metres, to the nearest whole metre.**

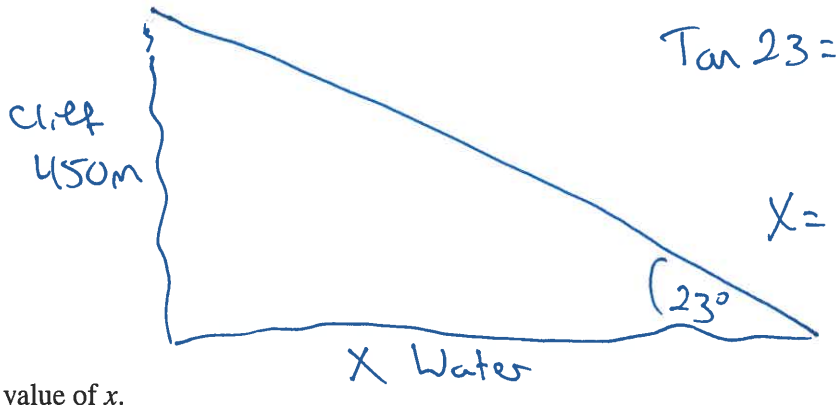
$\tan 28.3 = \frac{x}{1380}$ $\quad$ $x = 743$ m

~~(c)~~ The actual height of the cliff is 718 metres. Calculate the percentage error made by José when calculating the height of the cliff.

2. The height of a vertical cliff is 450 m. The angle of elevation from a ship to the top of the cliff is 23°. The ship is x metres from the bottom of the cliff.

(a) Draw a diagram to show this information.

Diagram:



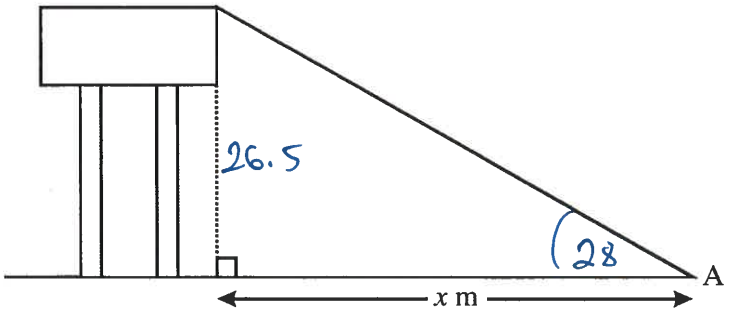
$\tan 23 = \frac{450}{X}$

$X = \frac{450}{\tan 23}$

(b) Calculate the value of x.

$X = 1060\text{m}$

3. The diagram shows a water tower standing on horizontal ground. The height of the tower is 26.5 m.



$\tan 28 = \frac{x}{26.5}$

From a point A on the ground the angle of elevation to the top of the tower is 28°.

(a) On the diagram, show and label the angle of elevation, 28°.

(b) Calculate, **correct to the nearest metre**, the distance x m.

14 m

Domain and Range

1. Which of the following sets of ordered pairs are functions? Write Function or Not a Function.

a $\{(1, 3), (2, 4), (3, 5), (4, 6)\}$

F

b $\{(1, 3), (3, 2), (1, 7), (-1, 4)\}$

NF

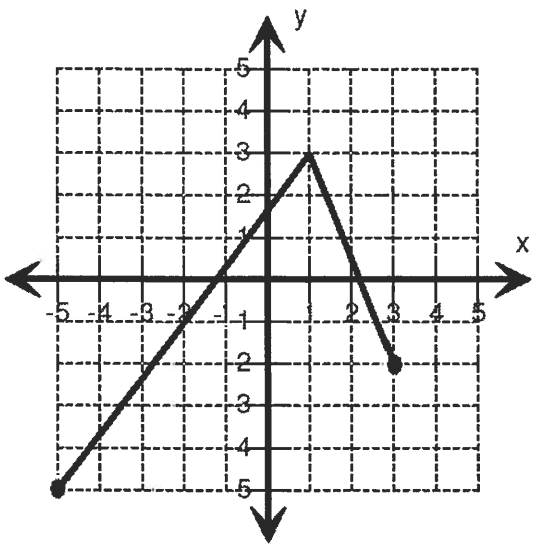
c $\{(2, -1), (2, 0), (2, 3), (2, 11)\}$

NF

d $\{(7, 6), (5, 6), (3, 6), (-4, 6)\}$

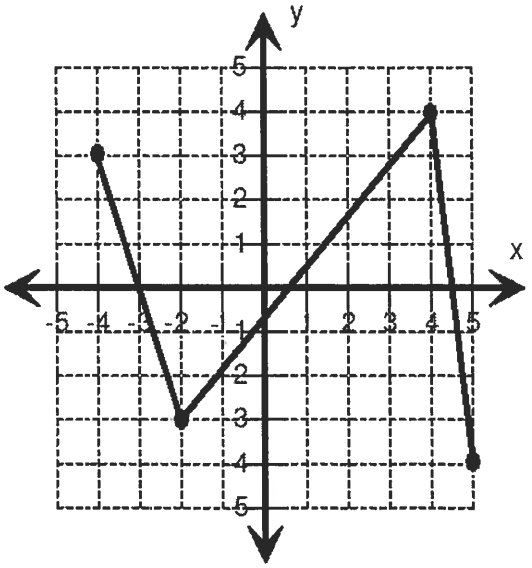
F

2. Find the **domain and range** for each of these relations. If they are not a function then **write NF.**



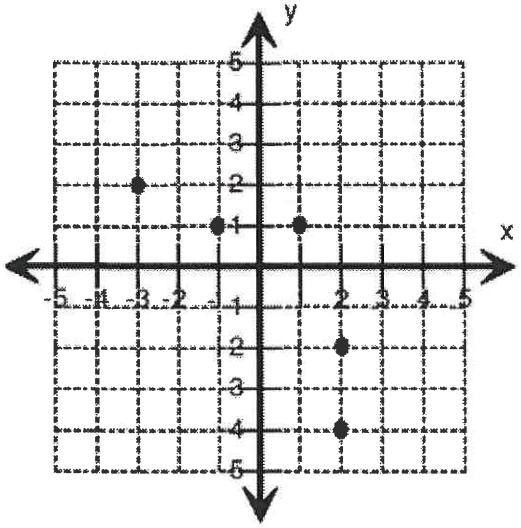
$D: \{-5 \leq x \leq 3\}$

$R: \{-5 \leq y \leq 3\}$



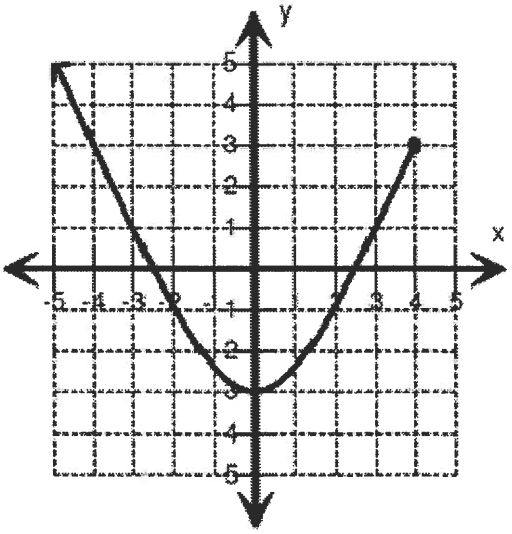
$D: \{-4 \leq x \leq 5\}$

$R: \{-4 \leq y \leq 4\}$



$D: \{-3, -1, 1, 2\}$

$R: \{-4, -2, 1, 2\}$



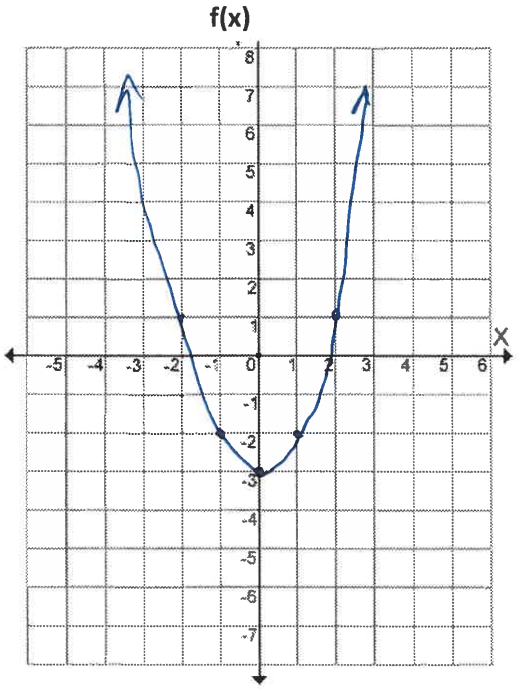
$D: \{x \leq 4\}$

$R: \{y \geq -3\}$

Functions

a) **Sketch** the following relation on the graph
Make sure to label the minimum or maximum.

$f(x) = x^2 - 3$



b) Find the Domain and Range.

D: $\{x \in \mathbb{R}\}$ R: $\{y \geq -3\}$

c) Find f(3).

$f(3) = (3)^2 - 3$

$f(3) = 6$

Evaluate the following expressions given the functions below:

$$g(x) = \frac{2x+3}{x-2}$$

a) $g(10)$

$$g(10) = \frac{2(10)+3}{10-2} = \frac{23}{8}$$

b) $g(-\frac{1}{2})$

$$g(-\tfrac{1}{2}) = \frac{2(-\tfrac{1}{2})+3}{-\tfrac{1}{2}-2} = -4/5$$

c) Find x if g(x) = -2

$$-2 = \frac{2x+3}{x-2}$$

$$-2x + 4 = 2x + 3$$

$$-4x = -1$$

$$x = \tfrac{1}{4}$$

$f(x) = 2x^2 + x - 7$

a) $f(x^2) = 2(x^2)^2 + (x^2) - 7$

$2x^4 + x^2 - 7$

b) $f(x-4) = 2(x-4)^2 + (x-4) - 7 = 2x^2 - 15x + 5$

$= 2(x^2 - 8x + 16) + x - 4 - 7$

c) $f(-2)$

$= 2(-2)^2 + (-2) - 7$

$= -1$

The value of a car in dollars, t years after its purchase is given by:
$V(t) = 25{,}000 - 3000t$

a) Find the original price of the car.

$V(0) = 25{,}000 - 3000(0)$

$V(0) = 25{,}000$

b) Find t when $V(t) = 10{,}000$ and explain what this represents.

$10{,}000 = 25{,}000 - 3000t$

$-25{,}000 \quad -25{,}000$

$\frac{-15{,}000}{-3000} = \frac{-3000t}{-3000}$

$t = 5$

It takes 5 years for the car to be worth $10,000

c) After ten years you decide to sell your car. You go to the dealership and they refuse to pay anything for the vehicle. Why might they be unwilling to buy the car? Justify your answer with calculations.

$V(10) = 25000 - 3000(10)$

$V(10) = -5000$

The car is worth a negative amount. Dealership would lose money.

An electrician charges $60 for getting to a job and $45 per hour he spends working on it.

a) Create a table of values for the cost the electrician charges against the time "t" hours he works for t = 0,1,2,3,4, and 5

Cost (dollars)	0	1	2	3	4	5
Time (hours)	60	105	150	195	240	285

b) Use the above information to create a cost function C(t).

$$C(t) = 45t + 60$$

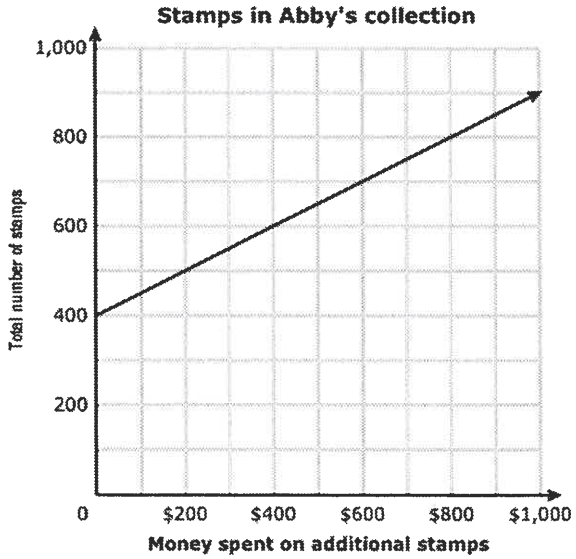
c) Use your function to determine the electrician's total cost for a job lasting 6 hours and forty five minutes.

$$C(6.75) = 45(6.75) + 60$$

$$C(6.75) = 363.75$$

This graph shows how the total number of stamps Abby has in her collection is related to the amount of money she spends on additional stamps.

2 Marks Each



a) How much money does Abby need to spend on new stamps in order to have a total of 500 stamps in her collection?

$200

b) If this trend continues and Abby spent $2000 dollars on stamps how many would she have in her collection?

1400 stamps

c) How many stamps did Abby initially have in her collection?

$400

PART B: **Problem-Solving**

For each of the following problems, you must

- Define a variable
- Write an equation
- Solve the equation
- Write a clear answer to the question

1. When a number is multiplied by negative two, the answer is thirty less than the same number. What is the number?

X = my number

$-2x = x - 30$

$-x \quad -x$

$\frac{-3x}{-3} = \frac{-30}{-3}$

$x = 10$

My answer is 10

2. Ahmed has a lot of coins in a bag. He has 4 times as many 25 cent coins as he does 50 cent coins. If Ahmed has 15 dollars total, how many of each type of coin does he have?

X = 50 cent coins

$0.5x + 4(0.25x) = 15.00$

Ahmed has 10 fifty cent coins and 40 25 cent coins

$\frac{1.5x}{1.5} = \frac{15.00}{1.5}$

$x = 10$

3. The Hammerit Tennis Club are holding a fund-raising dinner. Tickets for members cost 150 AED and tickets for guests cost 250 AED. There is a charge of 1200 AED to prepare the hall where the dinner will take place. All 50 members of the club will buy a ticket and the club wants to make a profit of 20,000 AED.

How many guests need to buy tickets to the dinner to make 20,000 AED?

members = m
guests = g

profit: $150(m) + 250(g) + 1200 = 20000$

$150(50) + 250g + 1200 = 20000$

$250g = 20000 - 1200 - 7500$

$g = \frac{20000 - 1200 - 7500}{250} = 45.2$

# of guests should be greater than 43

4. When a number is multiplied by negative two, the answer is thirty less than the same number. What is the number?

number = x

$-2(x) = x - 30$

$-2x = x - 30$

$30 = 3x$

$10 = x$

5. Eli is now one quarter of his father's age. In 5 years' time his age will be one third of his father's age. How old is Eli's father and Eli now?

	Eli	Father
Now	x	4x
5 years later	x+5	(4x+5)

$4x+5 = 3(x+5)$

$4x+5 = 3x+15$

$-3x \quad -5 \quad -3x \quad -5$

$x = 10$

Eli is 10 years old

his father is 40 years old

6. Four plus the **sum** of two consecutive odd integers is one less than three times the first odd integer. Find the first odd integer.

$4 + (x) + (x+2) = 3(x) - 1$

$6 + 2x = 3x - 1$

$7 = x$

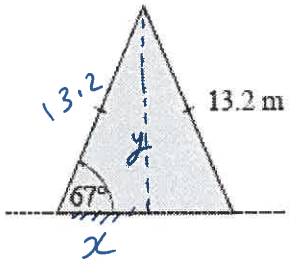
7. Five times the sum of a number and 3 is equal to 3 multiplied by 1 less than twice a number. Find the number.

$5(x+3) = 3(2x-1)$

$5x + 15 = 6x - 3$

$18 = x$

8.



An A-frame house has the shape of an isosceles triangle with base angles 67°. The oblique walls are 13.2 m long. Find:

a how wide the building is at ground level

b the height of the apex above the ground.

$\cos 67 = \frac{x}{13.2}$

$x = 13.2 \cos 67$

$x = 5.16\text{m} \Rightarrow \text{Base} = 2x = 2(5.16) = 10.3\text{m}$

Height:

$\sin 67 = \frac{y}{13.2}$

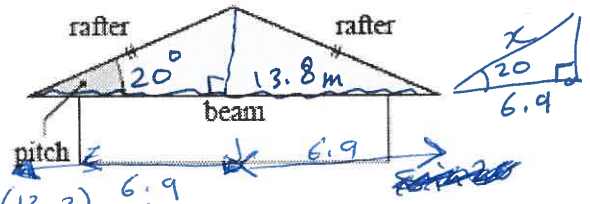
$y = 12.15$

9.

For the triangular roof truss illustrated, find:

a the length of a rafter if the beam is 13.8 m and the pitch is 20°

b the pitch of the roof if the rafter is 8.85 m long and the beam is 13.2 m long.



(a) $\cos 20 = \frac{6.9}{x}$

$x = \frac{6.9}{\cos 20} = 7.34$

(b) $\frac{1}{2}\text{beam} = \frac{1}{2}(13.2) = 6.6\text{ m}$

$\therefore \text{pitch} = \theta = \cos^{-1}\left(\frac{6.6}{8.85}\right)$

$= 83.5$

10.

At the local stationery shop, five pencils and six biros cost a total of €4.64, and seven pencils and three biros cost a total of €3.58. Find the cost of each item.

$5p + 6b = 4.64$

$7p + 3b = 3.58$

$p = €0.28$

$b = 0.54$

11.

Seven toffees and three chocolates cost a total of $1.68, whereas four toffees and five chocolates cost a total of $1.65. Find the cost of each of the sweets.

$7T + 3C = 1.68$

$4T + 5C = 1.65$

$T = \$0.15$

$C = \$0.21$

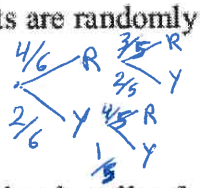
12.

A box contains 4 red and 2 yellow tickets. Two tickets are randomly selected one after the other from the box, *without replacement*.

a Display this information on a tree diagram.

b What is the probability that both are red?

c What is the probability that one is red and the other is yellow?



(a)

(b) $P(RR) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} = \frac{2}{5}$

(c) $P(RY) + P(YR)$

$= \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5}$

$\frac{8}{30} + \frac{8}{30} = \frac{16}{30} = \frac{8}{15}$

13.

If two consecutive integers have a sum of 173, find the numbers.

$x + (x+1) = 173$

$2x = 172$

$x = 86 \Leftarrow 1^{st}\ \#$

$x + 1 = 87 \Leftarrow 2^{nd}\ \#$

14.

If three consecutive integers add to 108, find the smallest of them.

$x + (x+1) + (x+2) = 108$

$3x + 3 = 108$

$3x = 105$

$x = 35$

The smallest # is 35.